Optimisation of the Cost of Lateritic Soil Stabilized with Quarry Dust

Dr. M. E. ONYIA
Department of Civil Engineering
University of Nigeria, Nsukka. Nigeria.

ABSTRACT: This paper proposes a model for optimising the production cost of loose lateritic soil stabilized with quarry dust using Scheffe's experimental design techniques on simplex lattice. The developed model uses a polynomial of second degree to express the behaviour of the components of the mixture in a simplex lattice. Weights of different components of the mixture generated from Scheffe's theory were used in arriving at the resulting cost model. The cost optimisation is based on current market prices of the mixture components, with a provision for future price fluctuation. A computer program for the cost optimisation based on the model is also developed. The cheapest mix proportion obtained by the model is 1:3:0.14 (Lateritic soil : Quarry dust : Water) with a total cost of 1768.26 Naira/m3. The model predictions were compared with the analytical results and found to be adequate at 5% significance level.

Keywords: CBR, Optimisation, Simplex lattice, Stabilization.

1.0 INTRODUCTION

The subgrade is the layer of natural soil prepared to receive the other layers of a road pavement. The loads on a pavement are ultimately supported by the subgrade and dispersed to the earth mass below. The subgrade will normally consist of undisturbed local material, or of soil excavated elsewhere and placed as fill. As there is usually frequent variation in subgrade soil type and the strength encountered along the line of a proposed road, base and sub-base material is necessary to spread the loading on the pavement over a sufficient area to avoid over-stress and consequent failure of the natural soil support. Lateritic soil is commonly used as base and sub-base material due, mainly, to its excellent load-bearing qualities. Engineers are therefore constantly searching for methods of improving its quality and optimizing its cost. Quarry dust, which is readily available as an industrial waste, has been proven to improve the bearing capacity of the soil, (**Indiramma et. al, 2016**).

This paper therefore aims at the development of a model for optimizing the cost of lateritic soil stabilized with quarry dust.

2.0 FORMULATION OF MODEL

H. Scheffe (1958) formulated a model for the assessment of the response of a particular characteristic of a mixture to variations in the proportions of its component materials. In his simplex lattice model, he considered experiments with mixtures in which the desired property (in this case, the production cost) depends on the proportion of the constituent materials present as

ISSN 2229-5518

atoms of the mixture. A simplex lattice can be described as a structural representation of lines joining the atoms of a mixture. It can be used as a mathematical space in model experiments involving mixtures by considering the atoms as the constituent components of the mixture, (**Akhnazarova et. al, 1982**).

When studying the components of a q-component mixture, which are dependent on the component ratio only, the factor space is a regular (q-1) simplex, and for the mixture, the following relationship holds, (**Scheffe, 1958**):

$$\sum_{i=1}^{q} X_i = 1 \qquad (1)$$

where $X_i \geq 0$ is the component concentration, q is the number of components.

For a 3-component mixture (q=3), the regular 2-simplex is an equilateral triangle, each with its interior. Each point in the triangle corresponds to a certain composition of the ternary system, and conversely each composition is represented by one distinct point. The vertices of the triangle represent pure substances, and the sides binary systems.

To describe the response surfaces in multi-component systems adequately, high degree polynomials are required, and hence, a great many experimental trials. The response is the property of mixture sought, and in this case it is the Cost of the mixture of lateritic soil, quarry dust and water. A polynomial of degree n in q variables has C_{q+n}^{n} coefficients, (Scheffe, 1958):

$$\hat{Y} = b_o + \sum_{1 \leq i \leq q} b_i x_i + \sum_{1 \leq i \leq j \leq q} b_{ij} x_i x_j + \sum_{1 \leq i \leq j \leq k \leq q} b_{ijk} x_i x_j x_k + \ldots\ldots\ldots + \sum b_{i1i2\ldots in}\, x_{i1} x_{i2} \ldots x_{in} \qquad (2)$$

The relationship $\sum_{i=1}^{q} X_i = 1$ enables the q^{th} component to be eliminated and the number of coefficients reduced to C_{q+n-1}^{n}.

H. Scheffe suggested to describe mixture properties by reduced polynomials obtainable from Eq.(2) subject to the normalization condition of Eq.(1) for the sum of independent variables.

For a ternary system, the polynomial has the general form:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 \qquad (3)$$

But $X_1 + X_2 + X_3 = 1 \qquad (4)$

Multiplying Eq.(4) by b_0 :

$$b_0X_1 + b_0X_2 + b_0X_3 = b_0 \qquad (5)$$

Multiplying Eq.(4) by X_1, X_2, and X_3 in succession gives:

ISSN 2229-5518

$$\left.\begin{array}{l} X_1^2 = X_1 - X_1X_2 - X_1X_3 \\ X_2^2 = X_2 - X_1X_2 - X_2X_3 \\ X_3^2 = X_3 - X_1X_3 - X_2X_3 \end{array}\right\} \qquad (6)$$

Substituting Eqs.(5) and (6) into Eq.(3), we obtain, after necessary transformations:

$$\hat{Y} = (b_0+b_1+b_{11})X_1 + (b_0+b_2+b_{22})X_2 + (b_0+b_3+b_{33})X_3 + (b_{12}-b_{11}-b_{22})X_1X_2 + (b_{13}-b_{11}-b_{33})X_1X_3 + (b_{23}-b_{22}-b_{33})X_2X_3 \qquad (7)$$

If we denote
$$\left.\begin{array}{l} \beta_i=b_0+b_i+b_{ii} \\ \beta_{ij} = b_{ij} - b_{ii} - b_{jj} \end{array}\right\} \qquad (8)$$

Then we arrive at the reduced second-degree polynomial in three variables:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_3 \qquad (9)$$

Thus the number of coefficients has reduced from ten to six.

In a more general form, the reduced second degree polynomial in q variables is

$$\hat{Y} = \sum_{1\leq i \leq q} \beta_i x_i + \sum_{1\leq i < j \leq q} \beta_{ij} x_i x_j \qquad (10)$$

The simplex lattice design provides a uniform scatter of points over the (q-1) simplex. The points form a (q, n)–lattice on the simplex, where q is the number of mixture components and n is the degree of the polynomial.

For a second degree polynomial, the (3, 2)-lattice is represented schematically in Fig.1

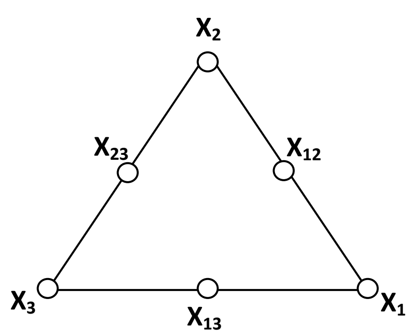

Fig. 1 – (3, 2)-lattice for a second-degree polynomial

We denote the components as:

X_1 = Proportion of lateritic soil in the mixture.

X_2 = Proportion of quarry dust in the mixture.

X_3 = Proportion of water in the mixture.

ISSN 2229-5518

As can be seen from Fig. 1, at any vertex of the triangle only one component of the mixture is present while at the boundary lines two components exist and the third is absent. Thus, points 1, 2 and 3 of the triangle have coordinates (1, 0, 0), (0, 1, 0) and (0, 0, 1) respectively. If we substitute the above lattice coordinates into Eq.(9), we obtain the coefficients of the second degree polynomial as:

$$\left.\begin{aligned} \beta_1 &= Y_1 \\ \beta_2 &= Y_2 \\ \beta_3 &= Y_3 \end{aligned}\right\} \qquad (11)$$

Also,

$$\left.\begin{aligned} \beta_{12} &= 4Y_{12} - 2Y_1 - 2Y_2 \\ \beta_{13} &= 4Y_{13} - 2Y_1 - 2Y_3 \\ \beta_{23} &= 4Y_{23} - 2Y_2 - 2Y_3 \end{aligned}\right\} \qquad (12)$$

Generally, the coefficients of the second-degree polynomial for a q-component mixture is given by:

$$\left.\begin{aligned} \beta_i &= Y_i \\ \beta_{ij} &= 4Y_{ij} - 2Y_i - 2Y_j \end{aligned}\right\} \qquad (13)$$

3.0 MATERIALS AND METHOD

The relation between the actual components [Z] and the pseudo-components [X] is, (**Eze et. al, 2010**):

$$[Z] = [A]\,[X] \qquad (14)$$

From the real components, a Z-matrix is formed whose transpose becomes the conversion of the factor from pseudo to real component.

Thus, if we select the first three mix ratios of our mixture components, (Lateritic Soil : Quarry Dust : Water), then the real component simplex will be as shown in Fig. 2 below.

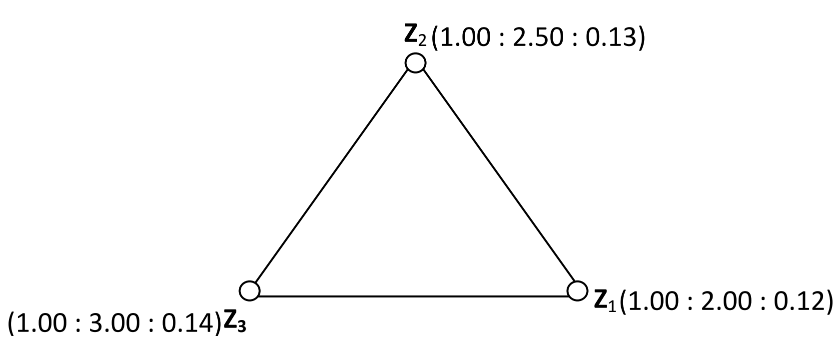

Fig. 2 – Real Component Simplex (only vertices shown)

Thus, [Z] = $\begin{bmatrix} 1.00 & 2.00 & 0.12 \\ 1.00 & 2.50 & 0.13 \\ 1.00 & 3.00 & 0.14 \end{bmatrix}$

And, $[Z]^T$ = $\begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 2.00 & 2.50 & 3.00 \\ 0.12 & 0.13 & 0.14 \end{bmatrix}$

Table 1 is a matrix table showing pseudo-components and real components for the (3, 2) - lattice. Note that a row in the real component side is obtained by multiplying $[Z]^T$ matrix by the corresponding row in the pseudo-component side of Table 1.

For example, for row 4 in Table 1 below:

$$\{Z\} = \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 2.00 & 2.50 & 3.00 \\ 0.12 & 0.13 & 0.14 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1.000 \\ 2.250 \\ 0.125 \end{bmatrix}$$

Table 1 – Matrix Table of Real and Pseudo-components for (3,2)-lattice Polynomial

S/N	**Pseudo-components**			**Response**	**Real components**		
	X_1	**X_2**	**X_3**		**Z_1**	**Z_2**	**Z_3**
1	1	0	0	Y_1	1.00	2.00	0.120

2	0	1	0	Y_2	1.00	2.50	0.130
3	0	0	1	Y_3	1.00	3.00	0.140
4	$\frac{1}{2}$	$\frac{1}{2}$	0	Y_{12}	1.00	2.25	0.125
5	$\frac{1}{2}$	0	$\frac{1}{2}$	Y_{13}	1.00	2.50	0.130
6	0	$\frac{1}{2}$	$\frac{1}{2}$	Y_{23}	1.00	2.75	0.135
Control points							
7	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	C_1	1.00	2.50	0.130
8	$\frac{1}{3}$	$\frac{2}{3}$	0	C_2	1.00	2.33	0.127
9	0	$\frac{1}{3}$	$\frac{2}{3}$	C_3	1.00	2.83	0.137
10	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	C_4	1.00	2.75	0.135
11	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	C_5	1.00	2.50	0.130
12	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	C_6	1.00	2.25	0.125

Legend: Z_1 = Lateritic Soil, Z_2 = Quarry Dust, Z_3 = Water

3.1 Laboratory CBR Test

The California Beaming Ratio (CBR) is an indicator of the mechanical strength of subgrade and base course beneath a road pavement. The test is therefore used for evaluating the suitability of subgrade and the materials used in sub-base and base courses, (**Ogundipe et. al, 2008**). The test is generally carried out in the laboratory on remoulded samples, as per BS 1377.

Lateritic soil for the test was collected from a construction site at the University of Nigeria, Nsukka Campus in Enugu State. The quarry dust was sourced from Ishiagu Quarry Plant in Ebonyia State. The soil, quarry dust and water were then mixed by weight as per the actual component ratios in Table 1. The mixture is subjected to CBR test as per **BS 1377 Pt. 9: 1990**. The costs in Table 6 were computed based on the market prices shown in Table 3 and the mix ratios and quantities shown in Table 5. Extra six test points (control points) were provided for validation of the model.

The physical properties of the mixture components are shown in Table 2.

Table 2 – Physical Properties of Materials

Component	Density (Kg/m^3)	Specific Gravity
Lateritic Soil	1900	2.4
Quarry Dust	1800	2.6
Water	1000	1.0

Source: Balamurugan et. al (2013)

Table 3 – Current Market Prices of Mix Components

S/N	Component	Unit Cost (Naira/Kg)
1	Laterite	1.25
2	Quarry Dust	0.47
3	Water	1.00

Table 4 – Optimal Values of Price Fluctuation Factor (PFF)

Number of years	PFF
Year 1	1.2326
Year 2	1.4638
Year 4	1.9300
Year 6	2.3939
Year 8	3.3260
Year 10	3.3260

Source: Nworu, G.E. and Unaeze, G.O. (1997)

Table 5 – Mix Quantities for $1m^3$ of Stabilized Laterite

S/N	Real Components (mix ratios)			Real Components Mix Quantities (kg/m^3)			CBR (%)
	z_1	z_2	z_3	Z_1	Z_2	Z_3	
1	1.00	2.00	0.120	827.43	1654.86	99.29	14.7
2	1.00	2.50	0.130	716.33	1790.81	93.12	15.4
3	1.00	3.00	0.140	631.52	1894.57	88.41	16.5
4	1.00	2.25	0.125	767.87	1727.71	95.98	15.1
5	1.00	2.50	0.130	716.33	1790.81	93.12	16.8
6	1.00	2.75	0.135	671.26	1845.96	90.62	18.2
Control points (7-12)							
7	1.00	2.50	0.130	716.33	1790.81	93.12	17.5
8	1.00	2.33	0.127	750.47	1748.59	95.31	15.4
9	1.00	2.83	0.137	657.92	1861.90	90.13	17.9
10	1.00	2.75	0.135	671.26	1845.96	90.62	17.9
11	1.00	2.50	0.130	716.33	1790.81	93.12	16.5
12	1.00	2.25	0.125	767.87	1727.71	95.98	16.1

Legend: Z_1 = Lateritic Soil, Z_2 = Quarry Dust, Z_3 = Water

S/N	Components Mix Quantities (kg/m^3)	Cost of Components ($Naira/m^3$)	Response Symbol	Total Estimated Cost ($Naira/m^3$)

Table 6 – Cost Estimate (in Naira) for 1m^3 of Stabilized Laterite

	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3		
1	827.43	1654.86	99.29	1034.29	777.78	99.29	Y_1	1911.36
2	716.33	1790.81	93.12	895.41	841.68	93.12	Y_2	1830.21
3	631.52	1894.57	88.41	789.40	890.45	88.41	Y_3	1768.26
4	767.87	1727.71	95.98	959.84	812.02	95.98	Y_{12}	1867.84
5	716.33	1790.81	93.12	895.41	841.68	93.12	Y_{13}	1830.21
6	671.26	1845.96	90.62	839.08	867.60	90.62	Y_{23}	1797.30
Control points (7-12)								
7	716.33	1790.81	93.12	895.41	841.68	93.12	C_1	1830.21
8	750.47	1748.59	95.31	938.09	821.84	95.31	C_2	1855.24
9	657.92	1861.90	90.13	822.40	875.09	90.13	C_3	1787.62
10	671.26	1845.96	90.62	839.08	867.60	90.62	C_4	1797.30
11	716.33	1790.81	93.12	895.41	841.68	93.12	C_5	1830.21
12	767.87	1727.71	95.98	959.84	812.02	95.98	C_6	1867.84

3.2DEVELOPMENT OF THE MODEL

The general form of Scheffe's reduced second-degree polynomial in q-variables is given by Eq.(10):

$$\hat{Y} = \sum_{1 \leq i \leq q} \beta_i x_i + \sum_{1 \leq i < j \leq q} \beta_{ij} x_i x_j \quad (15)$$

where,

$\beta_i = Y_i$, and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$

For a three-component mixture (i.e. (3, 2)-simplex lattice), q = 3

Therefore,

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_3 \quad (16)$$

Using the responses from Table 6 (column 9):

$\beta_1 = Y_1 = 1911.36$

$\beta_2 = Y_2 = 1830.21$

$\beta_3 = Y_3 = 1768.26$

$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2 = -11.78$

$\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3 = -38.40$

$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3 = -7.74$

Substituting these coefficients in Eq.(16), the required cost optimisation model of soil stabilized with quarry dust becomes:

$$\hat{Y} = (1911.36)X_1 + (1830.21)X_2 + (1768.26)X_3 + (-11.78)X_1X_2 + (-38.40)X_1X_3 + (-7.74)X_2X_3 \quad (17)$$

The predicted values from Eq.(17) are presented in Table 7, Column 7.

Table 7 – Computation of Response Values

S/N	Pseudo-components	Response Symbol	Estimated Cost	Predicted Cost (Response)

	X_1	X_2	X_3		(Naira/m^3)	(Naira/m^3)
1	1	0	0	Y_1	1911.36	1911.36
2	0	1	0	Y_2	1830.21	1830.21
3	0	0	1	Y_3	1768.26	1768.26
4	$\frac{1}{2}$	$\frac{1}{2}$	0	Y_{12}	1867.84	1867.84
5	$\frac{1}{2}$	0	$\frac{1}{2}$	Y_{13}	1830.21	1830.21
6	0	$\frac{1}{2}$	$\frac{1}{2}$	Y_{23}	1797.30	1797.30
Control points (7-12)						
7	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	C_1	1830.21	1831.29
8	$\frac{1}{3}$	$\frac{2}{3}$	0	C_2	1855.24	1854.64
9	0	$\frac{1}{3}$	$\frac{2}{3}$	C_3	1787.62	1787.19
10	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	C_4	1797.30	1796.98
11	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	C_5	1830.21	1830.45
12	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	C_6	1867.84	1868.19

3.3 VALIDATION OF THE MODEL (Test for Adequacy)

The model equation was tested to check whether the model results agree with the analytical results.

Let the statistical Null Hypothesis be denoted by H_o, and the alternative by H_I.

H_o: There is no significant difference between the analytical and the model predicted results.

H_I: There is significant difference between the analytical and the predicted results.

The Fisher Statistical Test was used to test the adequacy of the model. The model predicted values (Y_M) for the control points were obtained by substituting the corresponding pseudo-components (X_i) into the model equation, i.e. Eq. (17). The model results (Y_M) and the analytical results (Y_E) are shown in Table 7.

Table 8: F-statistics For the Control Points

S/N	Response	Y_E	Y_M	$Y_E – Y_{EA}$	$Y_M - Y_{MA}$	$(Y_E – Y_{EA})^2$	$(Y_M - Y_{MA})^2$
7	C_1	1830.21	1831.29	2.14	3.17	4.5796	10.0489
8	C_2	1855.24	1854.64	27.17	26.52	738.2089	703.3104
9	C_3	1787.62	1787.19	-40.45	-40.93	1636.2025	1675.2649
10	C_4	1797.30	1796.98	-30.77	-31.14	946.7929	969.6996
11	C_5	1830.21	1830.45	2.14	2.33	4.5796	5.4289
12	C_6	1867.84	1868.19	39.77	40.07	1581.6529	1605.6049
SUM		**10968.42**	**10968.74**			**4912.0164**	**4969.3576**

Legend: Y_E = Estimated Cost, Y_M = Model Cost, Y_{EA} = Average Estimated Cost

Y_{MA} = Average Model Cost, N = Number of points of observation.

Average Estimated Response, $Y_{EA} = \sum Y_E/N \quad = \frac{10968.42}{6} = 1828.07$

Average Model-predicted Response, $Y_{MA} = \sum Y_M/N \quad = \frac{10968.74}{6} = 1828.12$

where N is the number of responses.

Computing the variance for both analytical and model results:

$S_E^2 = \sum (Y_E – Y_{EA})^2/ (N-1) = \frac{4912.0164}{5} = 982.40$

$S_M^2 = \sum (Y_M – Y_{MA})^2/ (N-1) = \frac{4969.3576}{5} = 993.87$

The Fisher Test statistic factor is given by,

$F = S_M^2/S_E^2$, since S_M^2 is higher i.e. $F = \frac{993.87}{982.40} = 1.01$

From the standard F-distribution Table, for (N-1) degree of freedom the F-statistic, $F_{0.95}(5,5)$ = 5.05. Since this is higher than the calculated value of 1.01, we accept the Null Hypothesis. Therefore, the model equation is adequate.

Table 9 – Comparison of Analytical Costs and Predicted Costs

S/N	Response Symbol	Analytical Cost (Naira/m^3)	Predicted Cost (Naira/m^3)	Percentage Difference (%)
1	C_1	1830.21	1831.29	0.06
2	C_2	1855.24	1854.64	0.03
3	C_3	1787.62	1787.19	0.02
4	C_4	1797.30	1796.98	0.02
5	C_5	1830.21	1830.45	0.01
6	C_6	1867.84	1868.19	0.02
Average				**0.027**

4.0 CONCLUSIONS

A comparison of the predicted results with the analytical results shows that the average percentage difference is 0.027%, which is negligible. Also, the Fisher Test used in the statistical hypothesis showed that the developed model is adequate and reliable at 5% significance level for predicting the production cost of loose lateritic soil stabilized with quarry dust. The cheapest mix proportion obtained by the model is 1:3:0.14 (Lateritic soil : Quarry dust : Water) with a total cost of 1,768.26 Naira/m^3. The developed model can be used to determine the unit production cost of lateritic soil stabilized with quarry dust. Conversely, it can be used to obtain the mix proportions that can be afforded by a specified monetary budget. The model thus eliminates the arbitrary mixing of components and yields optimum mixtures, thereby minimizing costs.

REFERENCES

1. AKHNAZAROVA, S. and KATAROV, V. (1982), "*Experiment Optimization in Chemistry and Chemical Engineering*", Mir Publishers, Moscow.
2. BALAMURUGAN, G. and PERUMAL, P. (2013), "*Use of Quarry Dust to Replace Sand in Concrete – An Experimental Study*", International Journal of Scientific and Research Publications, Vol. 3, Issue 12.
3. BS 1377, "*Methods of Test of Soils for Civil Engineering Purposes*", British Standards Institution Publication.
4. EZE, J. C., IBEARUGBULEM, O. M., and ANYA, C. U. (2010), "*Optimisation of Aggregate Composition of Laterite/Sand Hollow Blocks Using Scheffe's Simplex Method*", International Journal of Engineering, Vol. 4 (4), 471-478.
5. INDIRAMMA, P. and SUDHARANI, C. (2016), "*Use of Quarry Dust for Stabilizing Expansive Soils*", International Journal of Innovative Research in Science, Engineering and Technology, Vol. 51.
6. NWORUH, G.E. and UNAEZE, G.O. (1997). "*Optimisation of Price Fluctuation Calculations*", Journal of Project Management, Vol. 1, Issue 1.
7. OGOAMAKA, P.M.C. (2004) "*Inferential Statistics for Research in Education and Social Sciences*", Peacewise Systems and Prints, Owerri.
8. OGUNDIPE and OLUMIDE MOSES (2008), "*Road Pavement Failure Caused by Poor Soil Properties along Aramoko-Ilesha Highway, Nigeria*" Journal of Engineering and Applied Sciences, Vol. 3, Issue 3.
9. ONUAMAH, P. N. and OSADEBE, N. N. (2015). "*Modeling and Optimization of the Compressive Strength of Lateritic Concrete Using Scheffe's Theory,*" International Organisation of Scientific Research Journal of Engineering (IOSRJEN), Vol.5 (5), 74-86.
10. OSADEBE, N. N., MBAJIOGU, C. C. and NWAKONUBI, T. U. (2007), "*An optimization Model Development for Laterized Concrete Mix Proportioning in Building Construction*", Nigerian Journal of Technology.
11. SCHEFFE, H. (1958), "*Experiments with Mixtures*", Journal of the Royal Statistical Society, Ser B., Vol. 20, 344-360.
12. SPIEGEL, M. R. and STEPHENS, L. J. (2011), "*Statistics*", McGraw Hill, New York.